Chaotic dynamics in the framework of the Vlasov-Nordheim equation

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ABSTRACT

In the framework of the time-dependent mean field theory, we solve the Vlasov-Nordheim equation and study the sensitive dependence of this highly non-linear equation on the initial conditions. We find that, when nuclear matter is inside the spinodal region, initially small differences between similar trajectories grow exponentially and produce strongly divergent paths resulting in a chaotic evolution. We focus on the fact that chaos spontaneously arises without putting any additional noise. We calculate the largest Lyapunov exponent for various initial densities. This analysis is performed for nuclear matter in two dimensions. It is argued that the mean field chaoticity can cause nuclear multifragmentation observed in intermediate energy heavy ion reactions.

1   Introduction

Intermediate energy heavy ion collisions offer a unique possibility to study nuclear matter under extreme conditions of density and temperature. In the energy range between 20 and 100 MeV/nucleon, the formation of hot nuclei and their successive decay, through emission of several intermediate mass fragments (IMF : $Z \geq 2$) with large fluctuations from event to event, has stimulated a huge debate on the main mechanism responsible of this phenomenon, commonly named "multifragmentation".

Both statistical models and dynamical simulations [1, 2] have been widely used providing suggestions on the possible paths followed by the nuclear system along the multifragmentation process. Surface instabilities or ring formation [2] have been recently considered as possible steps towards fragment formation. Quite surprisingly also the percolation model has been successfully used [3], suggesting a universal behavior typical of phase transitions and common to different fields. The onset of instability in the spinodal zone of the nuclear matter Equation of State (EOS) has been studied in ref. [4], by analyzing the corresponding linear

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[1]Talk given by A. Rapisarda.

response for imaginary frequency as a function of the wave number and of the density, even including a noise of Langevin-type to the collision integral in the BUU equation [5]. However already in the 80s, in one of the first attempts to describe multifragmentation Knoll *et al.* [6] performed numerical calculations in the framework of two-dimensional Time Dependent Hartree Fock (TDHF) equation which showed many of the features described by the recent BUU calculations [2]. In particular by evolving an ensemble of Slater determinants, they found a dependence on the initial conditions and a spontaneous symmetry breaking which lead to large fluctuation from event to event and in the charge or mass distributions. This scenario suggests a strong non-linear evolution, as recently suggested also in ref.[7, 8].

In order to clarify the mechanism of multifragmentation, we study in this contribution the dynamics of the Vlasov-Nordheim (VN) equation in the spinodal zone. We find unambiguously a strong non-linear evolution which implies a sensitive dependence on the initial conditions and a spontaneous symmetry breaking [8]. This means that the dynamics in the spinodal region is chaotic, although the equation is deterministic: similar trajectories can have a completely different evolution once they enter the spinodal region. We estimate also a Lyapunov exponent, which gives the mean rate of divergence between the trajectories in density space, in line with the usual analysis of dynamical systems [9]. We find that two-body collisions do not affect the onset of chaoticity in nuclear many-body systems. Finally we draw some conclusions. Further details can be found in ref. [8].

2 Theoretical framework

2.1 *The Vlasov-Nordheim equation*

The numerical analysis is performed by solving the Vlasov-Nordheim equation

$$\frac{\partial f}{\partial t} - \{H[f], f\} = I[f] \tag{1}$$

on a two-dimensional lattice, using the same code of ref.[5]. In eq. (1) $H[f]$ is the self-consistent effective one-body Hamiltonian, while $I[f]$ is the two-body Pauli blocked collision integral. The single particle phase space is divided into cells and nuclear matter is confined in a large torus with periodic boundary conditions. The side lengths of the torus are equal to $L_x = 51 \ fm$ and $L_y = 15 \ fm$. In order to achieve sufficient accuracy, very small cells having $\delta x = \frac{1}{3}$ fm and $\delta p = 40$ MeV/c are employed. The time integration step was chosen equal to 0.5 fm/c. We adopt a simplified Skyrme interaction for the effective one-body field which is averaged over the y-direction [5] and convoluted over a Gaussian function whose width is $\mu = 0.43$ fm. This is equivalent to introduce a finite effective range. We employ a Fermi momentum at saturation of $P_F = 260$ MeV/c, which gives a density in two dimensions equal to $\rho_0 = 0.55 \ fm^{-2}$.

2.2 *Definition of the largest Lyapunov exponent*

In the theory of dynamical systems, a measure of chaoticity is expressed in terms of Lyapunov exponents [9]. The largest Lyapunov exponent λ_∞ can be extracted by considering at a given time t the distance $d(t)$ between two trajectories, along an unstable direction in phase space, and define

$$\lambda(t) \;=\; \frac{\log(d(t)/d_0)}{t} \;, \tag{2}$$

where $d_0 = d(0)$. The average of this quantity along the trajectory gives the mean rate of exponential separation of neighbouring trajectories

$$\lambda_\infty \;=\; \lim_{t\to\infty}\lim_{d_0\to 0}\lambda(t) \tag{3}$$

In numerical applications one has to select a series of small values of d_0, and check that the corresponding values of $\lambda(t)$ stabilize around a definite value of $\overline{\lambda}$.

In order to characterize the system as a whole we have chosen as distance between two trajectories the difference in norm between their density profiles

$$d(t) \;=\; \sum_i |\rho_i^{(1)}(t) - \rho_i^{(2)}(t)|/N_c \;, \tag{4}$$

where the index i runs over the N_c cells in ordinary space, and $\rho_i^{(1)}$, $\rho_i^{(2)}$ are the densities in the cell i for the trajectories 1 and 2 respectively. The definition of eq. (4) is sufficient for the present analysis, although possible differences in momentum space are averaged out. It should include the contribution of all the unstable modes which dynamically grow up during the evolution.

3 Results and discussion

First we analyze the mean field dynamics neglecting two-body collisions, i.e. solving the left-hand side of eq.(1). We have initialized nuclear matter with a sinusoidal profile along the x-direction, characterized by a wave number $k = 2\pi(n_k/L_x)$ with $n_k = 5$ and a small amplitude equal to 1% of the local density. The local momentum distribution is assumed to be the one of a Fermi gas at a temperature $T = 3\ MeV$.

In fig.1a we show the evolution of the density profile for different times at normal density. The initial oscillation along the x coordinate is rapidly damped and, as fig.1b shows, the evolution of the Fourier transform of the density profile presents only one peak which disappears during the same time scale. Changing the initial density by a small amount, for example $d_0 = \frac{\Delta\rho}{\rho_0} = 10^{-2}$, one obtains exactly the same behavior and the evolution of the two events is indistinguishable. This is typical of the "regular" region of a dynamical system, and indicates the stability of the dynamics with respect to small perturbations.

On the contrary, if the density ρ is chosen well inside the spinodal region, $\rho = 0.5\rho_0$ in fig.2, the initial symmetry of the density profile, after a short time, is completely broken and fluctuations are rapidly enhanced.

The Fourier transform on the other hand shows the appearance of several new modes which where not excited initially. Moreover a small change in the initial density, see fig.2bis where $\rho = 0.51\rho_0$, produces an evolution which is completely different both in shape and in amplitude from that of fig.2. This is the typical behavior of a dynamical system in a chaotic region, where due to the non-linearity of the equation, very small initial perturbations are rapidly amplified and the distance between similar trajectories diverges exponentially.

In our case this behavior occurs even if $d_0 = \frac{\Delta\rho}{\rho_0} = 10^{-4}$, see fig.4. In general one finds an increase of the populated wave numbers together with a strong mixing as the evolution proceeds, despite the fact that only one wave number k is initially occupied. The final frequencies spectrum has little resemblance with the initial one. Note however that the initial mode is not completely damped, and some memory of it is still retained. We have checked that this general trend depends neither on the initial n_k nor on the shape of the initial profile.

In fig.3a it is displayed the time evolution of the distance between trajectories, as defined in eq.(4), normalized with respect to its initial value $d_0 = 10^{-4}$. We note that, while the distance remains constant in the regular zone as time goes on, it increases exponentially when the system starts in the spinodal region. In fig.3b $\lambda(t)$ as defined in eq.(2) is plotted vs. time. As one can see, λ tends to zero when dynamics is regular, indicating that the trajectories actually do not diverge between each other. On the contrary, λ rapidly converges to a value $\overline{\lambda}$ which is not zero in the unstable regions and unambiguously defines the largest Lyapunov exponent $\overline{\lambda}$. We note that our time scale for the onset of chaos is in agreement with other numerical simulations on instability growth [7, 6, 2] and with recent experimental results on multifragmentation [10].

We have checked that the results are stable with respect to the increase of the numerical precision. This is shown in fig.4, where the time evolution of λ is plotted for different values of the time integration step and initial distance. No appreciable variation of the results is obtained using a value of d_0 smaller than 10^{-4} or decreasing the time step. Therefore the divergence of the trajectories is an intrinsic dynamical property of the nuclear mean field in this region.

The inclusion of two-body collisions, performed by solving the VN equation as done in ref.[5], but without the Langevin term, does not modify substantially the dynamics, as also found independently in ref.[7]. This can be seen in fig.5, where λ is displayed vs. time for a density well inside the spinodal region. We note only a slight variation - about 10 %- of $\overline{\lambda}$ when including the collision term. In fact, since the dynamics is chaotic, the exponential divergence between trajectories can hardly be affected by the collisions, which have a small effect at low density because of the diluteness of the gas [5, 7].

Finally in fig.6 we show the extracted values of $\overline{\lambda}$ calculated at different densities. The values of $\overline{\lambda}$ increase from zero at $\rho = 0.66\rho_0$ to a a flat maximum around the density $\rho = 0.35\rho_0$. The curve reported in fig.6 , which is essentially

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independent from the initial k, clearly defines the nuclear matter spinodal region up to its upper limit.

It is worthwhile to compare the characteristic times $\tau_{MF} = \hbar/\overline{\lambda}$, which defines the time scale of the divergence between mean field trajectories, with the single particle characteristic time $\tau_{sp} = \hbar/E_F$, being E_F the Fermi energy at the given density. We have found that, for densities $\rho \leq 0.4\rho_0$, the divergence time is smaller than the single particle time, and therefore in this region the notion itself of mean field ceases to have any validity.

4   Conclusions

In the framework of a purely mean field theory, we have shown that the nuclear dynamics inside the EOS spinodal region is chaotic. Without adding any noise, and even without two-body collisions, spontaneous symmetry breaking occurs leading to large unpredictable density fluctuations. We characterize the chaoticity of the system through the largest Lyapunov exponent, for which we suggest a computational recipe. The latter does not depend on the particular choice of the initial conditions or the time integration step, but it is strongly related to the initial density. The small influence due to two-body collisions has been discussed, as well as the validity limits of the mean field theory. These results confirm quantitatively a scenario already suggested by other authors [6, 5, 7, 11] and can have important consequences for the whole picture of nuclear multifragmentation.

Generally speaking, a central collision between heavy ions at intermediate energy can be viewed as follows. In the first stage of the reaction nuclear matter gets compressed and a highly excited composite nuclear system is formed. During this phase, the dynamics is insensitive to small modifications of the initial conditions. Once the maximum compression is reached, nuclear matter starts to expand and can merge into the spinodal region. According to the results presented in this paper, at this point the dynamics becomes chaotic and the corresponding large density fluctuations dominate the cluster formation. The degree of chaoticity and fluctuations depends of course on the details of the dynamics which brings the reaction inside the spinodal region. We would like to stress that, since initially very close events follow completely different dynamical paths, an event-by-event description is strongly advised. Averaging over different theoretical paths can partially mask the non-linear evolution [12]. This scenario can support the behavior suggested by percolation model and at the same time disagree with an equilibrium statistical description since some memory of the initial mode seems to remain. Unfortunately, at the moment, the extension of these calculations to the case of collisions between heavy ions appears problematic even in two dimensions. A realistic theoretical simulation which is able to describe how the system enters into the spinodal region and how long it stays there is desirable. On the other hand an unambiguous experimental signature is needed.

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Figure captions

Fig.1   Evolution of the density profile (a) and its Fourier transform (b) at initial normal density, see text.

Fig.2   Evolution of the density profile (a) and its Fourier transform (b) at initial density $\rho = 0.5\rho_0$, see text.

Fig.2bis   Evolution of the density profile (a) and its Fourier transform (b) at initial density $\rho = 0.51\rho_0$, see text.

Fig.3   For three values of initial density the evolution of the distance between two trajectories (a) and the corresponding $\lambda(t)$ (b) are shown. The initial distance is $d_0 = 10^{-4}$, see text.

Fig.4   The quantity $\lambda(t)$ as a function of time changing d_0 and the time step. Using a value of d_0 smaller than 10^{-4} no change occurs.

Fig.5   Behavior of $\lambda(t)$ as a function of time with and without collision term in eq.(1).

Fig.6   The Lyapunov exponent $\overline{\lambda}$ vs. ρ/ρ_0 at temperature T=3 MeV, see text.

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